Single-phase Blending of Liquids

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Previous investigations of mixing operations were concerned, for the most part, with determining what power was required to achieve a designated degree of mixing with various mixing devices (*1*). Quillen (*2*), Hixson and Smith (*3*), and Rushton (*4*) have commented on the scarcity of published data on blending rates and mixing time, and Hixson and Smith (*3*) have suggested the need for further work on mass transfer rates between miscible liquids.

The present investigation, assuming that the time required to achieve the desired degree of mixing was an important factor of any mixing problem, developed a method to measure this time interval, or mixing time, for a complete degree of mixing. The effects on this mixing time of tank dimensions, fluid properties, diameter and velocity of mixing jets, and the diameter and rotational speed of square-pitch marine propellers were investigated. General equations were developed to correlate all the measured mixing times and the above-mentioned variables. Calculation of mixing time by the development and use of the *mixing-time factor* as a function of the Reynolds number is the key to correlation of all the data. Use of the mixing-time factor in mixing-time calculations is analogous to use of the friction factor in friction-head-loss calculations. The appropriate mixing-time factor is determined from a Reynolds number and then combined with the geometry and dynamics of the blending system in order to calculate the mixing time.

The equations which were developed are believed to be generally applicable to the blending of any miscible Newtonian liquids having equal densities and viscosities. *It is important to note that* small differences in these liquid properties were deliberately excluded or minimized in this work. Rushton (*7*) has noted that density differences can greatly affect mixing time, and the equations should not be applied to blending of liquids having appreciably large differences in densities or viscosities until the effects of these variations are further explored.

It was found that liquid blending is a function of the flux of momentum of the mixing device, rather than the power input. In fact, the same mixing result in a given blending operation can be achieved with various power inputs by changing the diameter and speed of a propeller mixer or the diameter and velocity of a mixing jet. A large, low-speed propeller or jet can produce equivalent mixing results with less power than is required with a smaller, higher speed propeller or jet.

DEFINITION OF TERMS

1. *Degree of Mixing*: the uniformity of composition which is desired in the finished mix or blend. To be precisely stated, it should include the size of sample to be considered, the limits of variation from the average value which it is desired to maintain for any one ingredient or property, and the required percentage of samples which must fall within these desired limits.
2. *Terminal Mixing*: a specific degree of mixing achieved at the time when uniformity of composition in the specified sample size, within the precision of the instrumention used, is no further changed by additional mixing. This is something less than absolutely complete mixing, which implies homogeneity even to the molecular level.
3. *Mixing Time*: the time required from the start of a mixing or blending operation to the time when terminal mixing is achieved.
4. *Turbulent Mixing*: that type of mixing which results when the Reynolds number of a jet is greater than 2×10^3 or when the Reynolds number of a propeller is greater than 10^4.
5. *Incipient Laminar Mixing*: that type of mixing which results when the Reynolds number of a jet is between 10^2 and 2×10^3 or when the Reynolds number of a propeller is between 10^2 and 10^4.

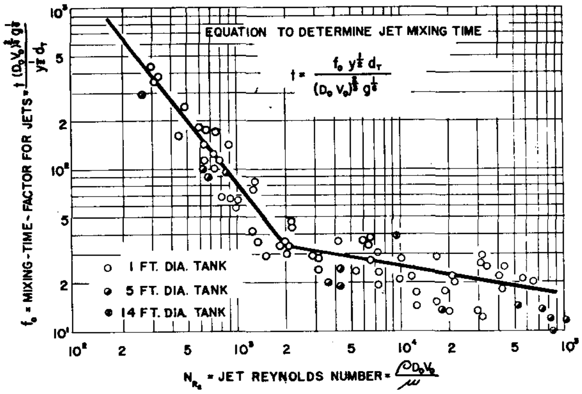

Fig. 1. Correlation of all data to determine jet-mixing-time factor (f_0).

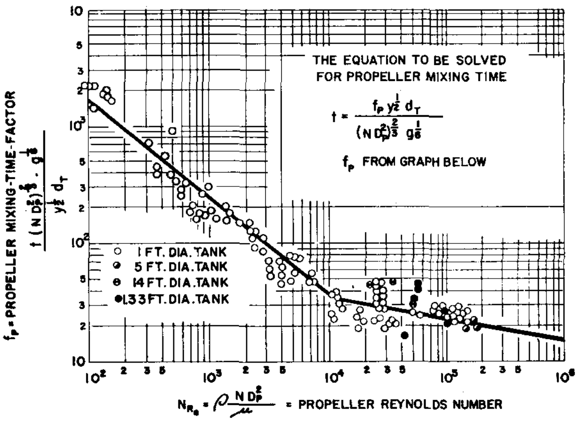

Fig. 2. Correlation of all data to determine propeller-mixing-time factor (f_p).

EQUIPMENT AND MEASUREMENTS

Laboratory mixing data were taken in cylindrical vessels 1 and 5 ft in diameter. The smaller tank was made of glass, and the larger of steel painted white on the inside. The number of test runs made in the 1-ft. tank was 220; in the 5-ft. tank, 110. Other data at a factory installation were obtained from a steel tank of 14-ft. diameter and height; six test runs were made in this tank. The batch liquids tested separately were water, glycerol, and vegetable oils.

For the laboratory data, terminal mixing was determined by visual interpretation; for the factory data, terminal mixing of hardened and unhardened oils was determined by the equilibrium point of the iodine values.

The visual method of determining terminal mixing in the laboratory was as follows. The tank was filled with a liquid and a saturated solution of phenolphthalein was added. Sodium hydroxide solution, $2N$, was then added until the pH reached the range (7.9 to 8.1) where the red color of the indicator became evident. A small amount of $2N$ hydrochloric acid was then added until the faint pink color just disappeared. With the batch mix at equilibrium, a measured amount of lye was added which made the contents a deep red. The mixing equipment was then put in motion long enough to achieve equilibrium. Then an exactly equivalent amount of the neutralizing acid was added instantaneously; at the same instant a stop watch was started (the acid having been premixed with glycerine so as to be quite close to the density and viscosity of the batch mix into which it was placed). When the very last wisp of red color was observed to disappear, the watch was stopped and the elapsed time recorded. The degree of mixing which was attained at this instant cannot be easily defined mathematically but the method is more complete and more stringent than any known method of analyzing samples and establishing a standard deviation. The fact that the very last wisp of red color *which can be visually observed* is isolated and located at a different position in the tank for different flux of momentum strongly supports this argument.

The method of iodine value for determination of terminal mixing in factory operations was as follows. A small amount of hardened oil was added to a large amount (15,000 gal.) of unhardened oil. The mixing equipment was put in motion and a stop watch started simultaneously. Samples (60 cc.) of the mixed oils were drawn at 30-sec. intervals each from three stratagically located sample ports for a period of 20 to 30 min. The sample ports were kept freely, flowing throughout a test run. The samples were analyzed in the laboratory for iodine value, and a plot of iodine value vs. time indicated the terminal point at which there was no variation greater than the possible error of the test instrument (±0.5 I.V. index).

The velocity at the jet exhaust was calculated from measured flow rate and jet diameter. Flow rates were measured with standard orifice or with a rotameter.

It should be pointed out that the mixing time reported for all laboratory tests (with the 1- and 5-ft.-diam. tanks) is that associated with the large batch already in equilibrium of motion, to which a small amount of tracer fluid is added. In the factory tests (with a 14-ft.-diam. tank) this was not possible, and hence the mixing time is that associated with the batch mixing started from standstill.

The viscosities in the higher ranges were measured with a Brookfield viscometer; in the lower ranges of viscosities, the Rich-Roth Ultra-viscoson was used.

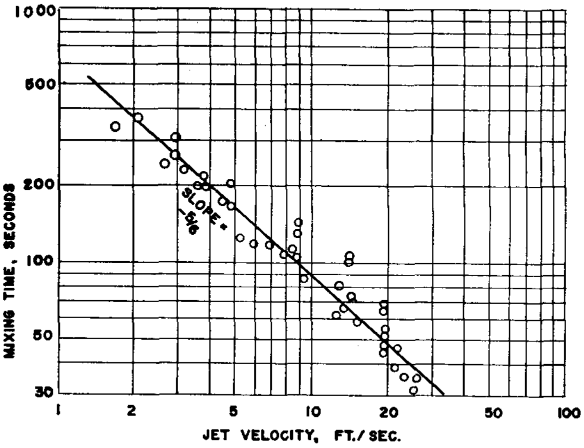

Fig. 3. Slope for exponent of jet velocity (turbulent regime).

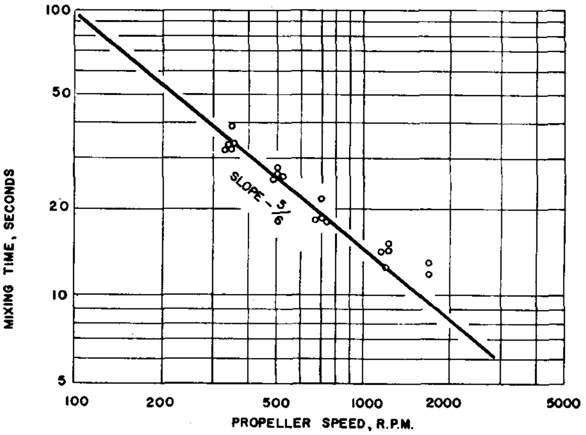

Fig. 4. Slope for exponent of propeller rotational speed (turbulent regime).

PROCEDURE

Two basic types of mixers were investigated: jet mixers and three-blade square-pitch propellers. In the jet system fluid was recirculated from the tank to the jet. In both propeller and jet testing, the mass of fluid in the mixing vessel for any one test run was constant. The seven independent variables assumed to affect mixing time (t) were

Jets	Propellers
1. d_t = tank diameter	d_t = tank diameter
2. y = depth of liquid in the tank	y = depth of liquid in the tank
3. D_0 = jet diameter	D_p = propeller diameter
4. V_0 = jet velocity	N = rotational speed of propeller
5. μ = batch liquid viscosity	μ = batch liquid viscosity
6. ρ = batch liquid mass density	ρ = batch liquid mass density
7. g = gravitational acceleration	g = gravitational acceleration

that is:

$t = f(d_t, y, D_0, V_0, \mu, \rho, g)$	$t = f(d_t, y, D_p, N, \mu, \rho, g)$

For each device all variables except one were held constant for a series of runs. The exponent of this variable was determined by plotting mixing time as a function of this variable on log-log paper. Typical series are shown in Figures 3, 4, and 5. Each plot approximated a straight line, and accordingly all were assumed to be straight lines.

The exponents of four of the seven numbered independent variables listed above were in each of the four cases (turbulent and incipient laminar regimes for both jets and propellers) determined in the foregoing manner. The exponents of the remaining three variables were then unique-

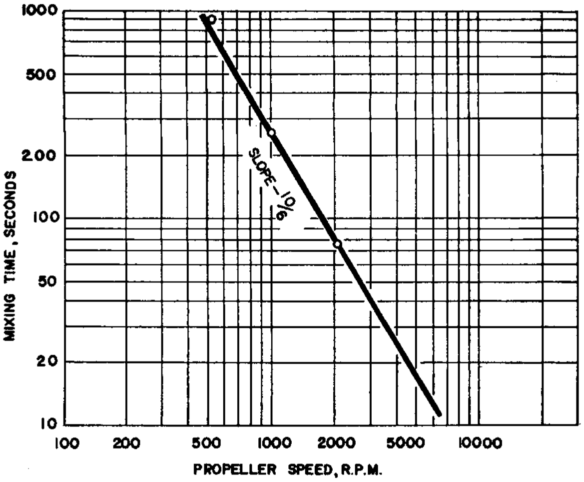

Fig. 5. Slope for exponent of propeller rotational speed (laminar regime).

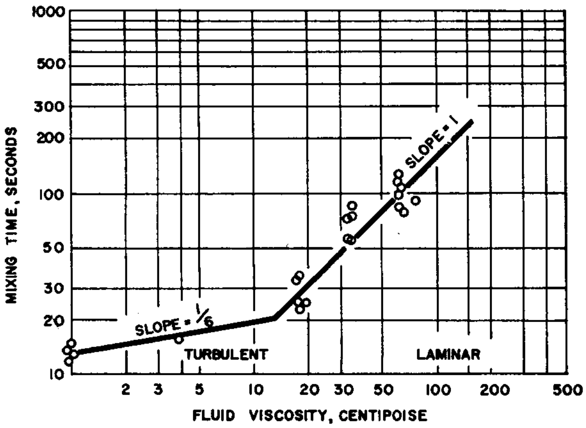

Fig. 6. Slopes for exponent of fluid viscosity (propeller, laminar, and turbulent regimes).

ly fixed by dimensional analysis. This having been done, the exponent of the fifth variable, viscosity, was experimentally determined. Agreement of the exponent of viscosity by both methods was considered a good check on the accuracy of the investigation.

It was only after a limited number of test runs had been made that it became apparent that there were distinct turbulent and incipient laminar regimes of mixing. Consequently, exponents were determined for jets and propellers in both regimes, as illustrated by Figure 6 for viscosity. The ranges of variables investigated were as follows:

Variable	Range
Viscosity (μ)	from 0.5 to 400 centipoise
Tank diameter (d_t)	from 6 in. to 14 ft.
Depth of liquid (y)	from 6 in. to 14 ft.
Ratio of liquid depth to tank diameter (y/d_t)	from ¼ to 2/1
Propeller diameter (D_p)	from 1 to 22 in.
Rotational speed (N)	from 200 to 2,200 rev./min.
Blade-tip velocity (V_p)	from 2 to 36 ft./sec.
Jet velocity (V_0)	from 1 to 50 ft./sec.
Jet diameter (D_0)	from 1/16 to 1-½ in.
Ratio of tank diameter to propeller diameter (d_t/D_p)	from 3/1 to 14/1
Ratio of tank diameter to jet diameter (d_t/D_0)	from 45/1 to 180/1

RESULTS

Representative data are compiled in Tables 1 and 2. The results are best summarized by writing the equations which express mixing time as a function of all variables under investigation. These equations may be written in the following form:

$$t = C_1 \frac{y^{1/2} d_t}{(V_0 D_0)^{5/6}} \cdot \left(\frac{\mu}{\rho}\right)^{1/6} \cdot \frac{1}{g^{1/6}}$$

(jet mixing, turbulent regime) (1)

TABLE 1. SUMMARY OF DATA

Representative values only, jet mixing data

Run	D_0, ft.	V_0, ft./sec.	D_0V_0	$(D_0V_0)^{2/3}$	$g^{1/6}$	t, sec.	$t(D_0V_0^{2/3})g^{1/6}$	$y^{1/2}$, ft.$^{1/2}$	D_T, ft.	$\frac{t(D_0V_0)^{2/3}g^{1/6}}{y^{1/2}D_T}$ f_0 = mixing-time factor	ρ, (lb.)(sec.^{2})/ft.^{4}	μ, (lb.)(sec.)/ft.^{2}	$\frac{\rho}{\mu}$	$\frac{\rho}{\mu} D_0V_0$ R_0 = Reynolds number
11	0.052	19.2	1.00	1.00	1.78	63	112	2.24	5.0	10.0	1.94	1.68×10^{-5}	1.15×10^5	1.15×10^5
12	0.052	26.1	1.35	1.22	1.78	47	103	2.24	5.0	9.2	1.94	1.68×10^{-5}	1.15×10^5	1.55×10^5
23	0.078	14.2	1.10	1.07	1.78	57	109	2.09	5.0	10.4	1.94	1.68×10^{-5}	1.15×10^5	1.26×10^5
25	0.078	8.2	0.64	0.74	1.78	114	150	2.24	5.0	13.3	1.94	1.68×10^{-5}	1.15×10^5	7.4×10^4
32	0.078	2.1	0.16	0.30	1.78	243	130	1.97	5.0	13.2	1.94	1.68×10^{-5}	1.15×10^5	1.8×10^4
35	0.078	9.4	0.73	0.81	1.78	67	96	1.66	5.0	11.6	1.94	1.68×10^{-5}	1.15×10^5	8.4×10^4
39	0.052	19.2	1.00	1.00	1.78	61	109	1.91	5.0	11.4	1.94	1.68×10^{-5}	1.15×10^5	1.15×10^5
21	0.021	34.4	0.73	0.81	1.78	71	102	2.09	5.0	9.7	1.94	1.68×10^{-5}	1.15×10^5	8.4×10^4
45	0.0312	46.4	1.46	1.29	1.78	61	140	2.16	5.0	13.0	1.94	1.68×10^{-5}	1.15×10^5	1.68×10^5
65	0.0835	15.0	1.25	1.16	1.78	91	188	1.94	5.0	19.4	1.94	0.67×10^{-3}	2.9×10^3	3.64×10^3
73	0.0835	15.0	1.25	1.16	1.78	303	630	1.94	5.0	65	1.94	$0..0 \times 10^{-2}$	6.5×10^2	8.1×10^2
81	0.0835	9.8	0.82	0.88	1.78	1,800	1,780	1.94	5.0	295	1.94	0.58×10^{-2}	3.3×10^2	2.7×10^2
94	0.0835	10.0	0.84	0.89	1.78	600	950	1.94	5.0	98	1.94	0.27×10^{-2}	7.2×10^2	6.1×10^2
101	0.0104	6.7	0.07	0.17	1.78	1,200	365	1.05	0.94	370	1.94	0.40×10^{-3}	4.9×10^3	3.4×10^2
103	0.0104	6.7	0.07	0.17	1.78	560	170	1.05	0.94	174	1.94	0.21×10^{-3}	9.2×10^3	6.4×10^2
106	0.0104	6.7	0.07	0.17	1.78	146	45	1.05	0.94	45	1.94	0.63×10^{-4}	3.1×10^4	2.2×10^3
123	0.0104	27.2	0.28	0.43	1.78	41	32	1.05	0.94	32	1.94	1.8×10^{-5}	1.1×10^5	3.1×10^4
137	0.0104	28.0	0.29	0.44	1.78	27	21	1.05	0.94	21	1.94	9.5×10^{-5}	2×10^5	5.8×10^4
176	0.0052	31.0	0.16	0.30	1.78	23	13	1.05	0.94	13	1.94	1×10^{-5}	1.9×10^5	3×10^4
176A	0.125	27.0	3.4	2.30	1.78	480	1,980	3.6	14	39.5	1.75	0.63×10^{-3}	2.8×10^3	9.5×10^3

$$t = C_2 \frac{y^{1/2} d_t}{(V_0 D_0)^2} \cdot \left(\frac{\mu}{\rho}\right)^{8/6} \cdot \frac{1}{g^{1/6}}$$

(jet mixing, incipient laminar regime) (2)

$$t = C_3 \frac{y^{1/2} d_t}{(N D_p^2)^{5/6}} \cdot \left(\frac{\mu}{\rho}\right)^{1/6} \cdot \frac{1}{g^{1/6}}$$

(propeller mixing, turbulent regime) (3)

$$t = C_4 \frac{y^{1/2} d_t}{(N D_p^2)^{10/6}} \cdot \frac{\mu}{\rho} \cdot \frac{1}{g^{1/6}}$$

(propeller mixing, incipient laminar regime (4)

The foregoing equations are then rearranged algebraically so as to include the Reynolds number as a separate grouping in each equation, thus:

$$t = C_1 \frac{y^{1/2} d_t}{(N_{re})^{1/6}} \cdot \frac{1}{(V_0 D_0)^{4/6}} \cdot \frac{1}{g^{1/6}}$$

(jet mixing, turbulent regime) (5)

$$t = C_2 \frac{y^{1/2} d_t}{(N_{re})^{8/6}} \cdot \frac{1}{(V_0 D_0)^{4/6}} \cdot \frac{1}{g^{1/6}}$$

(jet mixing, incipient laminar regime) (6)

$$t = C_3 \frac{y^{1/2} d_t}{(N_{re})^{1/6}} \cdot \frac{1}{(N D_p^2)^{4/6}} \cdot \frac{1}{g^{1/6}}$$

(propeller mixing, turbulent regime) (7)

$$t = C_4 \frac{y^{1/2} d_t}{(N_{re})} \cdot \frac{1}{(N D_p^2)^{4/6}} \cdot \frac{1}{g^{1/6}}$$

(propeller mixing, incipient laminar regime) (8)

It is important to note that after the Reynolds number has been grouped in each of the Equations (5), (6), (7), and (8), the remaining terms in all equations are identical. This provides the basis for correlation.

While the Reynolds number provides the device by which correlation is made, reexamination of Equations (1), (2), (3), and (4) for the flux of momentum proves to be a valuable aid in understanding the physical significance of some of the variables in the mixing operation. Dimensional observation shows that the units of flux of momentum (in the lb.-force, ft., sec. system of units) is simply pounds force. Or, formally stated (*6*), the flux of momentum $\rho A_0 V_0 \cdot V_0$ across an area A_0 has the same effect on a system as a force of magnitude $\rho A_0 V_0 \cdot V_0$ acting at the center of A_0. It can be shown in the general case of a jet that flux of momentum is proportional to the product $(D_0 V_0)^2$. Further, combination of this and results published by Rushton (*1*) shows that for propeller mixing in the turbulent regime the flux of momentum is proportional to $(N D_p^2)^2$; and for propeller mixing in the laminar regime the flux of momentum is proportional to $(N D_p^2)^{4/3}$.

Next it is interesting to note that in all the basic equations [(1) to (4)] the portion of the corresponding momentum terms $(V_0 D_0)$ and $(N D_p^2)$ to some exponent is common in the denominator of all equations. If any specific blending problem *in which tank dimensions and fluid properties are constant* is considered, the proportionalities can then be written

For jets / For propellers

$$t \propto \frac{1}{(V_0 D_0)^{5/6}} \qquad t \propto \frac{1}{(N D_p^2)^{5/6}}$$

(turbulent)

$$t \propto \frac{1}{(V_0 D_0)^2} \qquad t \propto \frac{1}{(N D_p^2)^{10/6}}$$

(laminar)

On substitution of momentum, these proportionalities become

For jets / For propellers

$$t \propto \frac{1}{(M_0)^{5/12}} \qquad t \propto \frac{1}{(M_p)^{5/12}}$$

(turbulent)

$$t \propto \frac{1}{(M_0)} \qquad t \propto \frac{1}{(M_p)^{5/4}}$$

(laminar)

This indicates that the time required to complete mixing is inversely proportional to the appropriate exponential value of the flux of momentum in any particular mixing problem under consideration where tank dimensions and fluid properties are held constant. If only the fluid properties were held constant, the mixing time would be inversely proportional to the appropriate exponential value of the flux of momentum and directly proportional to the square root of the tank volume [i.e., $t \propto Vol/(M_0)^n$]. Any combination of jet diameter and velocity which produces the same flux of momentum in the same flow regime (turbulent or incipient laminar) will then produce the same mixing time; but each combination will have a different kinetic energy flux and, hence, power requirement. Similarly for propellers, a given mixing time in a given flow regime (turbulent or incipient laminar) can be achieved by an infinite number of combinations of propeller speeds and diameters, each with a different power requirement. For either propeller or jet, the combinations of large diameters with low speeds or velocities will require less power than will the combinations of smaller diameters with higher speeds or velocities. One of the underlying conclusions based on the test data is that blending time is dependent on the flux of momentum into the mixing vessel and, as far as the time required to complete mixing is concerned, the same result can be achieved by large mass flow at low velocity as by small mass flow at high velocity, provided that the product of the two (flux of momentum) is the same. The data also show that as power (flux of kinetic energy) of the mixing device is held constant, but is expended through larger diameter jets at lower velocities (or larger diameter propellers at lower revolutions per

TABLE 2. SUMMARY OF DATA

Representative values only, propeller-mixing data

Run	D_p, ft.	N, rev./sec.	ND_p^2	$(ND_p^2)^{2/3}$	$g^{1/6}$	t, sec.	$t(ND_p^2)^{2/3}g^{1/6}$	$y^{1/2}$, ft.$^{1/2}$	D_T, ft.	$\frac{t(ND_p^2)^{2/3}g^{1/6}}{y^{1/2} d_T}$ f_p = mixing-time factor	ρ, (lb.)(sec.^{2})/ft.^{4}	μ, (lb.)(sec.)/ft.^{2}	$\frac{\rho}{\mu}$	$\frac{\rho N D_p^2}{\mu}$ N_{Re} = Reynolds number
231	0.166	30.0	0.83	0.91	1.78	15	24.2	0.97	0.94	25.6	1.94	1.76×10^{-5}	1.1×10^5	9.15×10^4
236	0.166	25.0	0.69	0.78	1.78	17	23.6	0.97	0.94	26.0	1.94	1.76×10^{-5}	1.1×10^5	7.6×10^4
248	0.166	8.4	0.23	0.38	1.78	27	18.2	0.97	0.94	20.0	1.94	1.76×10^{-5}	1.1×10^5	2.5×10^4
256	0.166	6.2	0.17	0.32	1.78	34	19.4	0.97	0.94	21.3	1.94	1.76×10^{-5}	1.1×10^5	1.9×10^4
269	0.166	30.6	0.84	0.89	1.78	25	39.5	0.97	0.94	43.5	2.01	0.38×10^{-3}	5.3×10^3	4.45×10^3
297	0.166	32.0	0.88	0.92	1.78	80	131.0	0.97	0.94	144.0	2.22	1.3×10^{-3}	1.2×10^3	1.49×10^3
304	0.166	30.6	0.84	0.89	1.78	110	174.0	0.97	0.94	192.0	2.32	2.1×10^{-3}	1.1×10^3	9.10×10^2
314	0.166	16.3	0.45	0.59	1.78	270	284.0	0.97	0.94	313.0	2.26	1.7×10^{-3}	1.3×10^3	5.85×10^2
320	0.166	34.5	0.95	0.97	1.78	75	129.0	0.97	0.94	142.0	2.12	1.7×10^{-3}	1.3×10^3	1.23×10^3
336	0.125	37.2	0.58	0.70	1.78	195	244.0	0.97	0.94	268.0	2.12	1.4×10^{-3}	1.5×10^3	8.7×10^2
345	0.125	29.5	0.46	0.60	1.78	240	256.0	0.97	0.94	282.0	2.12	1.4×10^{-3}	1.5×10^3	6.9×10^2
349	0.125	17.3	0.27	0.42	1.78	660	492.0	0.97	0.94	543.0	2.12	1.4×10^{-3}	1.5×10^3	4.1×10^2
350	0.083	30.6	0.21	0.35	1.78	1325	822.0	0.97	0.94	905.0	2.12	1.4×10^{-3}	1.5×10^3	3.2×10^2
350A	1.83	6.3	21.0	7.50	1.78	90	1,200	3.6	14.0	23.8	1.75	0.63×10^{-3}	2.8×10^3	5.9×10^4
350B	1.00	7.7	7.7	3.90	1.78	300	2,090	3.7	14.0	40.2	1.75	0.63×10^{-3}	2.8×10^3	2.2×10^4

minute), blending time continues to decrease rather than reaching a minimum, or optimum, value. In short, this indicated that there was no optimum power or head-flow relationship as far as the dynamics of the blending system was concerned, provided of course that the tendency to vortex was sufficiently restrained to keep the propeller always totally submerged. Only factors external to the blending, such as economics of power costs vs. capital costs of equipment, or pump efficiencies and pipe friction losses of a recirculating system, will define an optimum arrangement.

The flux-of-momentum concept can explain why it has been observed (*4*) in a number of cases that more horsepower is required of a jet to do the same mixing job as a propeller. The propeller has usually been a device of large diameter and low rotational speed; whereas the jet has commonly been a device of small diameter and high velocity.

The foregoing principles offer a real aid to reaching an economic balance between first costs and operating costs in selecting an agitator for a specific blending operation. A large, low-speed propeller can do the required job with lower power but usually higher first cost than a small, high-speed propeller. Balancing operating-power costs against equipment costs should make it possible to select the most economical mixer for the blending job at hand.

Analysis

It did not seem obvious that the results should have been correlated by the Reynolds number. At Reynolds numbers considerably higher than those explored in this investigation, it has been suggested that the slope of the curve Reynolds number vs. mixing-time-factor may decrease or approach zero, an indication that at extremely high Reynolds numbers (greater than 10^6), mixing time might be nearly, if not completely, independent of fluid properties. If this should be the case, the data in this investigation would indicate that the mixing time would then be dependent only on the volume of the batch and the propeller diameter–velocity product (which is momentum at a constant density).

The investigation could have been extended to include a wider range of Reynolds numbers, but the time involved did not permit this extension. It is possible that at very low Reynolds numbers (less than 100) the exponents of the variables may have different values. There may be a second transition point to a regime of more completely laminar mixing which would indicate that the range which is described in this report is properly termed *incipient* laminar mixing. Further investigation of the mixing-time-factor at both higher and lower Reynolds numbers by other members in the field would be most welcome

The coincidence that the regime of jet mixing changes from laminar to turbulent at the same critical Reynolds number associated with pipe flow indicates that, for the ratios of tank size to jet size investigated and for the viscosity ranges explored, a turbulent exhaust from the jet is propagated in turbulent fashion throughout the entire tank (*5, 8*). This may not be true for ranges of variables beyond those explored; however, the ranges which were explored cover those most commonly encountered in plant installations and hence should be valid for application.

The data correlated in Figures 1 and 2 show deviations as high as 60%, which are attributed primarily to (1) errors in visual interpretation of end point and (2) errors in visual interpretation of the initial neutral point at the beginning of each run. For most test runs there were two or more observers who attempted to determine independently the instant at which the end point was reached. These independent observations showed that while differences in answers as high as 60% might occasionally occur, the majority of answers agreed within 10%, and a number of observations were identical.

Alternative methods of measuring the end point seemed impractical. A colorimetric instrument focused on one area of the tank would have given erroneously low blending times for all laminar tests, since the point of last residual color shifted in position within the tank from one run to another and also during a single run. For turbulent mixing, colorimetric measurements might have equaled or exceeded the accuracy of visual observations, but its added complexity made this alternative unattractive.

The accuracy of grab sampling, as exemplified by the few oil-blending tests in the data reported on the 14-ft.-diam. tank, is limited in smaller tanks and is cumbersome in any case. In the oil-blending tests referred to, the samples taken represented 1.15 p.p.m. by volume of the entire batch. When each millionth part of the batch was like all others, the batch was considered to be uniformly blended for the purpose of that operation. The data obtained by this method on a tank of this size were included to show how closely they came to correlating with the data taken in the 1- and 5-ft. laboratory tanks using color indicator as described.

The mixing-time data obtained from plant tests (14-ft.-diam. tank) have higher values than the laboratory tests (1- and 5-ft. tanks), as expected, since the plant data were concerned with the mixing time beginning from a standstill; whereas laboratory data were all taken beginning from an equilibrium of motion. The time penalty for beginning a mixing operation from standstill was not investigated in detail, but several tests indicated it to be roughly 25% more than the time required for mixing beginning from an equilibrium of motion.

The effect of using varying amounts of lye and acid per unit mass of batch liquid was investigated, and it was found that the variation has a considerable effect on the mixing time up to a critical value, after which the increase of lye and acid per unit mass of liquid has no measurable effect on the mixing time. The critical value so determined was found to be approximately 1 cc. of 2*N* lye or 2*N* acid/gal. of batch liquid. All mixing-time data recorded were taken at conditions above this critical level. Subsequent testing also showed that the mixing time is not affected by the relative volumes of acid layer to base layer.

In the jet tests the effect of jet location was extensively investigated. Provided that the jet is placed so that (1) it does not induce a total swirl to the liquid in the tank or (2) it does not feed directly back into the suction of the recirculation system, it was found that the position of the jet was not particularly critical and that the variations in mixing time so induced were not detectable within the accuracy of the mearuring technique. The obvious point noted was that change of position of the jet merely changes the position of the last part of the tank fluid to be mixed. In propeller mixing, investigation showed that location of the propeller was much more critical. In all propeller tests the propeller was located so that the tendency toward vortexing was sufficiently restrained to keep the propeller always totally submerged in the liquid. All data recorded were concerned with optimum location of the propeller, that is, the location at which the mixing time was least. Failure to locate this point precisely, particularly in those regions where location appears to be most critical, may also account for some of the scattering of the data. No baffles were used in any tests.

In turbulent mixing, the exponents for the corresponding variables of jets and propellers are identical. In incipient laminar mixing, the exponents of the velocity terms and the kinematic voscosity were found to be slightly less for propellers than for jets. [See Equations (1) to (4).] This indicates that in incipient laminar mixing, as kinematic viscosity increases, the mixing time with propellers is decreased more rapidly than for jets; however, in a compensating negative effect, as the speed of the propeller increases, the mixing time with propellers is decreased less rapidly than for a corresponding velocity increase in a jet.

The plots of the individual variables, Figures 3, 4, and 5 for example, were all assumed to be straight lines. Since this assumption may not necessarily be valid, it could also account for some of the scattering of data evident in Figures 1 and 2.

It should be pointed out that there was no attempt made to maintain dimensional or geometric similarity between various tank sizes. To have done so would have resulted in data for special cases or would have presumed the effect of some common dimensionless groups. That the results were correlated by investigating one independent variable at a time, without the necessity of maintaining geometric similarity, was an indication that the work was fundamental and suitable for application to the general case.

In correlation of the data with the Reynolds number, the dimensionless group in Equations (5) to (8) remaining after the Reynolds number to the appropriate exponent is extracted is in each case

$$\frac{t(V_0D_0)^{4/6}\cdot g^{1/6}}{y^{1/2}d_t} \tag{9}$$

and

$$\frac{t(ND_p^2)^{4/6}\cdot g^{1/6}}{y^{1/2}d_t} \tag{10}$$

If these dimensionless groups are called the *mixing-time factors* and assigned the value f_0 and f_p, for jets and propellers respectively, one may then write

For jets

$$t = \frac{f_0 y^{1/2} d_t}{(V_0D_0)^{4/6}\cdot g^{1/6}} \tag{11}$$

For propellers

$$t = \frac{f_p y^{1/2} d_t}{(ND_p^2)^{4/6}\cdot g^{1/6}} \tag{12}$$

Application

To determine the mixing time for any case, then, Figure 1 or 2 is entered with the appropriate Reynolds number in order to determine the mixing-time factor f_0 or f_p. The value of f having been found, Equations (11) and (12) (which are repeated on Figures 1 and 2) may be solved for the mixing time. The graphs and equations in Figures 1 and 2 can be used with any system of consistent units.

Figures 1 and 2 thus can be used to determine the mixing time required to complete any given mixing or blending of a single-phase Newtonian system once the following information is established: liquid properties, tank dimensions, propeller or jet dimensions, and pumping rate for jets or rotational speed for propellers. Or conversely, for design purposes, the discharge velocity for jets or the rotational speed for propellers can be determined from the graphs once, in addition to the foregoing data, the required mixing time is established.

Having determined liquid properties, jet size and velocity (or propeller size and rotational speed), the manufacturers of pumps (or mixers, as the case may be) can then supply accurate data on horsepower requirements. This, then, bridges the gap between time and horsepower required to complete a simple mixing or blending operation. Figures 1 and 2 are applicable for any general case to be considered, provided that none of the following limits covered by this investigation are exceeded: d_t/D_0 is not greater than 180/1, y/d_t is not greater than 2/1 or less than 1/3, d_t/D_p is not greater than 14/1, viscosity is not greater than 400 centipoise, and fluids are miscible Newtonian with nearly equal densities and viscosities.

NOTATION

		M, L, T dimensions M = lb.-mass, L = ft., T = sec.	F, L, T dimensions F = lb.-force
A_0	= jet cross-sectional area	L^2	L^2
A_p	$= \frac{\pi D_p^2}{4}$ = propeller cross-sectional area	L^2	L^2
C	= numerical coefficient	Dimensionless	Dimensionless
D_0	= jet diameter	L	L
D_p	= propeller diameter	L	L
d_t	= tank diameter	L	L
f_0	= mixing-time factor for jets	Dimensionless	Dimensionless
f_p	= mixing-time factor for propellers	Dimensionless	Dimensionless
g	= gravitational acceleration	$\frac{L}{T^2}$	$\frac{L}{T^2}$
M_0	$= \omega_0 V_0$ = flux of momentum (jets)	$\frac{M}{T}\cdot\frac{L}{T} = \frac{ML}{T^2}$	F
M_p	$= \omega_p V_p$ = flux of momentum (propellers)	$\frac{M}{T}\cdot\frac{L}{T} = \frac{ML}{T^2}$	F
N	= propeller revolutions per second	$1/T$	$1/T$
N_{re}	$= \frac{\rho V_0 D_0}{\mu}$ = Reynolds number at jet	Dimensionless	Dimensionless
N_{re}	$= \frac{\rho N D_p^2}{\mu}$ = Reynolds number at propeller	Dimensionless	Dimensionless
P_0	$= \omega_0 V_0^2$ = flux of kinetic energy (power)	$\frac{ML^2}{T^3}$	$\frac{FL}{T}$
t	= mixing time	T	T
V_0	= velocity through jet	$\frac{L}{T}$	$\frac{L}{T}$
y	= height of fluid in tank	L	L
ρ	= mass density of fluid	$\frac{M}{L^3}$	$\frac{FT^2}{L^4}$
μ	= viscosity of fluid	$\frac{M}{LT}$	$\frac{FT}{L^2}$
ω	$= \rho A_0 V_0$ = mass flow rate	$\frac{M}{T}$	$\frac{FT}{L}$

Subscripts

0 = jet
p = propeller

ACKNOWLEDGMENT

The authors wish to acknowledge the contributions made by other members of the Procter and Gamble Company. Also, recognition is accorded to persons outside the company who rendered helpful suggestions and criticisms.

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Presented at A.I.Ch.E. Detroit meeting